2 UNIT MATHEMATICS FORM VI

Time allowed: 3 hours (plus 5 minutes reading) **Exam date:** 8th August 2001

Instructions:

All questions may be attempted.
All questions are of equal value.
All necessary working must be shown.
Marks may not be awarded for careless or badly arranged work.
Approved calculators and templates may be used.
A list of standard integrals is provided at the end of the examination paper.

Collection:

Each question will be collected separately.
Start each question in a new 4-leaf answer booklet.
If you use a second booklet for a question, place it inside the first. Don't staple.
Write your candidate number on each answer booklet.

QUESTION ONE (Start a new answer booklet)

Marks

$\boxed{1}$ (a) Convert $\frac{4\pi}{5}$ to degrees.

$\boxed{1}$ (b) Write down a primitive of $\sec^2 5x$.

$\boxed{2}$ (c) The line $5x - ky = 7$ passes through the point $(1, 1)$. Find the value of k.

$\boxed{2}$ (d) Differentiate $y = 5x^3 - 2x + 9$ with respect to x.

$\boxed{2}$ (e) Express $\dfrac{4}{\sqrt{3}-1}$ with a rational denominator in simplest form.

$\boxed{2}$ (f) Find the exact value of $\tan\frac{\pi}{3} + \tan\frac{\pi}{4}$.

$\boxed{2}$ (g) Solve $|x - 1| = 11$.

QUESTION TWO (Start a new answer booklet)

(a) Differentiate the following with respect to x:

Marks

$\boxed{2}$ (i) $x^2 e^x$,

$\boxed{2}$ (ii) $\ln(3x - 2)$,

$\boxed{2}$ (iii) $\sin^2 x$.

$\boxed{2}$ (b) Find a primitive function of $(3x - 4)^6$.

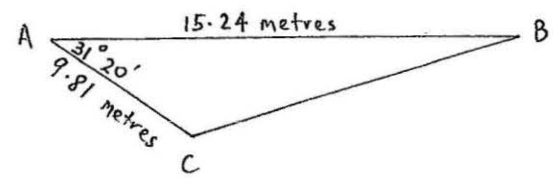
(c) Evaluate the following definite integrals:

$\boxed{2}$ (i) $\displaystyle\int_1^2 6x^2\,dx$,

$\boxed{2}$ (ii) $\displaystyle\int_0^{\frac{\pi}{2}} \sin 2x\,dx$.

QUESTION THREE (Start a new answer booklet)

(a)



Marks

$\boxed{2}$ (i) Find the length of BC correct to the nearest centimetre.

$\boxed{2}$ (ii) Find the area of $\triangle ABC$ correct to the nearest square metre.

(b) Consider the geometric series $1 - \dfrac{1}{3} + \dfrac{1}{9} - \cdots$.

$\boxed{1}$ (i) Explain why the series has a limiting sum.

$\boxed{1}$ (ii) Find the limiting sum.

$\boxed{3}$ (c) Find the equation of the tangent to the curve $y = \ln x$ at the point $(e, 1)$.

(d) If α and β are roots of the equation $x^2 + 8x + 11 = 0$, find:

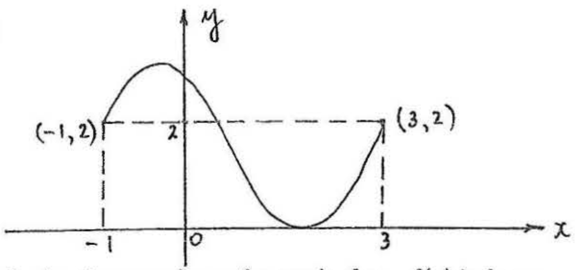
$\boxed{1}$ (i) $\alpha + \beta$,

$\boxed{1}$ (ii) $\alpha\beta$,

$\boxed{1}$ (iii) $\alpha^2 + \beta^2$.

QUESTION FOUR (Start a new answer booklet)

(a)



In the diagram above, the graph of $y = f(x)$ is drawn.

Marks

$\boxed{1}$ (i) Sketch the graph of $y = f(x) + 2$.

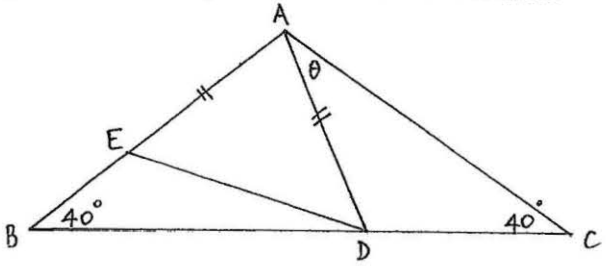
$\boxed{1}$ (ii) Given that $\displaystyle\int_{-1}^{3} f(x)\,dx = \frac{15}{2}$, evaluate $\displaystyle\int_{-1}^{3} (f(x) + 2)\,dx$.

(b) A particle moves in a straight line so that its displacement x metres at time t seconds is given by $x = 2t^3 - t^2$.

[2] (i) At what times is the particle at rest?

[2] (ii) How far does the particle travel between these times?

(c)



In the diagram above, $\triangle ABC$ is isosceles with $\angle B = \angle C = 40°$, and $AD = AE$. Let $\angle DAC = \theta$.

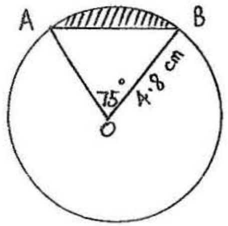
[1] (i) Explain why $\angle ADB = 40° + \theta$.

[2] (ii) Find an expression for $\angle DAE$ in terms of θ.

[3] (iii) Show that $\angle EDB = \frac{1}{2}\theta$.

QUESTION FIVE (Start a new answer booklet)

(a)



In the diagram above, O is the centre of a circle of radius 4·8 centimetres, and $\angle AOB = 75°$.

Marks

[2] (i) Find the exact length of arc AB.

[2] (ii) Find the exact area of the sector AOB.

[2] (iii) Find the area of the minor segment that has been shaded. Give your answer correct to three decimal places.

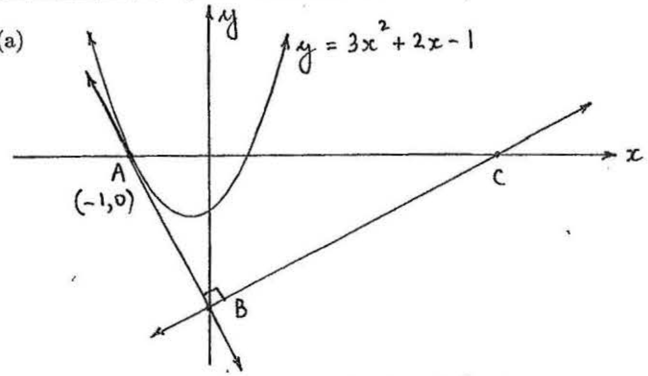
[2] (b) (i) Solve $\tan x = -3$ for $0 \le x \le 2\pi$. Give your answer in radians correct to three decimal places.

[2] (ii) On the same diagram, sketch graphs of $y = \tan x$ and $y = -3$ for $0 \le x \le 2\pi$.

[2] (iii) How many solutions are there to the equation $\tan x = -3$ in the domain $-2\pi \le x \le 2\pi$?

QUESTION SIX (Start a new answer booklet)

(a)



In the diagram above, the graph of $y = 3x^2 + 2x - 1$ and the tangent to the curve at the point $A(-1, 0)$ are drawn.

Marks

[2] (i) Show that the equation of the tangent is $y + 4x + 4 = 0$.

[1] (ii) Show that the tangent meets the y-axis at $B(0, -4)$.

[2] (iii) Find the equation of the line that passes through B and which is perpendicular to the tangent.

[1] (iv) Show that this line meets the x-axis at the point $C(16, 0)$.

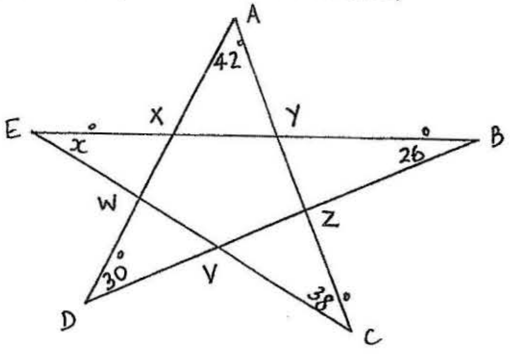
[2] (v) Find the area of $\triangle ABC$.

[4] (b) The region bounded by the curve $y = \tan x$ and the x-axis from $x = 0$ to $x = \frac{\pi}{4}$ is rotated about the x-axis. The volume of the solid formed is given by $V = \pi \int_0^{\frac{\pi}{4}} \tan^2 x \, dx$. Use Simpson's rule with the three function values $x = 0$, $\frac{\pi}{8}$ and $\frac{\pi}{4}$ to approximate the volume. Give your answer correct to three decimal places.

QUESTION SEVEN (Start a new answer booklet)

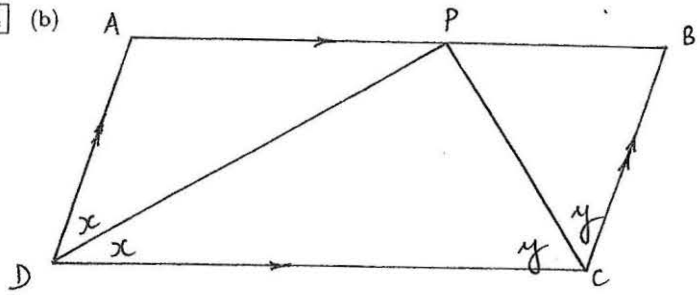
Marks

[3] (a)



Find the value of x. (Give reasons.)

[4] (b)



In the diagram above, $ABCD$ is a parallelogram. The point P lies on AB and it is known that $\angle ADP = \angle CDP = x$ and $\angle BCP = \angle DCP = y$.
Prove that $2AD = AB$. (Give reasons.)

[1] (c) (i) Write down the discriminant of $5x^2 - 2x + k$.

[2] (ii) For what values of k does $5x^2 - 2x + k = 0$ have real roots?

[2] (d) Solve $\log_e 16 = 2\log_e x$.

Exam continues next page ...

QUESTION EIGHT (Start a new answer booklet)

(a) Kerry deposits \$1500 into a superannuation fund on January 1st 2001. He makes further deposits of \$1500 on the first of each month up to and including December 1st 2010. The fund pays compound interest at a monthly rate of 0·75%. In each of the following questions give your answer to the nearest dollar.

Marks

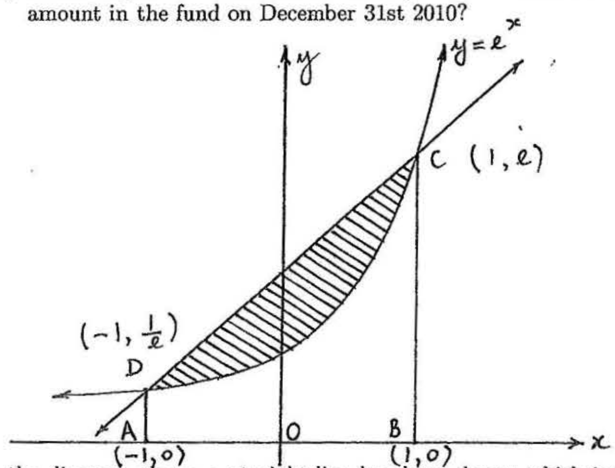
[1] (i) How much is in the fund on January 31st 2001?

[1] (ii) How much is the first \$1500 deposit worth on December 31st 2010?

[3] (iii) Form a geometric series and hence determine the total amount in the fund on December 31st 2010.

[2] (iv) If each deposit was increased to \$1600, what difference does it make to the total amount in the fund on December 31st 2010?

(b)



In the diagram above, a straight line has been drawn which intersects with $y = e^x$ at the points $C(1, e)$ and $D(-1, \frac{1}{e})$. The point A has coordinates $(-1, 0)$ and B has coordinates $(1, 0)$. The area between the curves has been shaded.

[2] (i) Show that the area of the trapezium $ABCD$ is given by $\dfrac{e^2+1}{e}$.

[3] (ii) Hence, or otherwise, find the exact area between the curves.

Exam continues overleaf ...

QUESTION NINE (Start a new answer booklet)

(a) The value $V of a car is given by the formula $V = Ce^{-kt}$, where C and k are constants and t is the time measured in years. Michael bought a car on June 30th 2001 which cost $65 000 and which was worth $55 000 after one year.

Marks

$\boxed{2}$ (i) Evaluate the constants C and k.

$\boxed{1}$ (ii) Find the value of the car after 5 years. Give your answer correct to the nearest dollar.

$\boxed{2}$ (iii) In which year will the value of the car fall below half its cost price for the first time?

(b) Concrete is pumped from a truck into a building foundation. The rate $R\,\mathrm{m}^3$/hour at which the concrete is flowing is given by the expression $R = 9t^2 - t^4$ for $0 \le t \le 3$, where t is the time measured in hours after the concrete begins to flow.

$\boxed{1}$ (i) Find the rate of flow at time $t = 2$.

$\boxed{1}$ (ii) Explain why t is restricted to $0 \le t \le 3$.

$\boxed{3}$ (iii) Find the maximum flow rate of concrete.

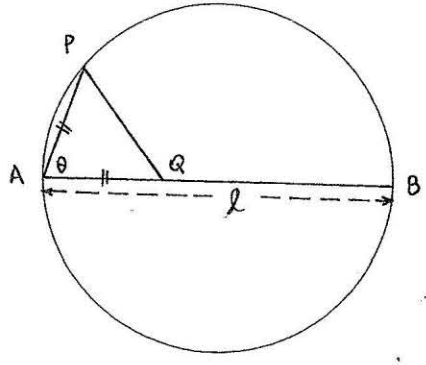
$\boxed{2}$ (iv) When the concrete begins to flow, the foundation has $1000\,\mathrm{m}^3$ already in place. Find an expression for the amount of concrete in the foundation at time t.

QUESTION 10 IS ON THE NEXT PAGE.

Exam continues next page ...

QUESTION TEN (Start a new answer booklet)

(a)



In the diagram above, P is a point on the circle with diamater $AB = \ell$. The point Q is on the diameter such that $AP = AQ$. Let $\angle PAQ = \theta$ and let S be the area of $\triangle PAQ$.

Marks

$\boxed{3}$ (i) Show that $S = \dfrac{\ell^2}{2}\cos^2\theta\sin\theta$.

$\boxed{3}$ (ii) Find the maximum area of $\triangle APQ$ as P moves along the circumference of the circle.

$\boxed{2}$ (b) (i) Find A and B such that $\dfrac{1}{(x-1)x} = \dfrac{A}{x-1} + \dfrac{B}{x}$.

$\boxed{2}$ (ii) Let $S_n = \dfrac{1}{1\times 2} + \dfrac{1}{2\times 3} + \dfrac{1}{3\times 4} + \cdots + \dfrac{1}{(n-1)n}$.

Show that $S_n = 1 - \dfrac{1}{n}$.

$\boxed{2}$ (iii) Hence or otherwise evalute $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(n-1)n}$.

JNC

Grammar 2001

SOLUTIONS

QUESTION 1

a) 144° ✓

b) $\frac{1}{5}\tan 5x + c$ ✓

c) $5 - k = 7$ ✓

$\therefore k = -2$ ✓

d) $\frac{dy}{dx} = 15x^2 - 2$ ✓✓ (1 each)

e) $\frac{4}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{4(\sqrt{3}+1)}{2}$ ✓

$= 2(\sqrt{3}+1)$ ✓

f) $\sqrt{3}+1$ ✓✓ (1 each)

g) $|x-1| = 11$

$x - 1 = 11$ or $x - 1 = -11$

$\therefore x = 12$ or -10 ✓✓

QUESTION 2

a) (i) $\frac{dy}{dx} = 2xe^x + x^2e^x$ ✓✓

$= xe^x(2+x)$ (not necessary)

(ii) $\frac{dy}{dx} = \frac{3}{3x-2}$ ✓✓

(iii) $\frac{dy}{dx} = 2\sin x.\cos x$ ✓✓

b) $\frac{(3x-4)^7}{21}$ ✓✓

c) (i) $\int_1^2 6x^2\,dx = \left[2x^3\right]_1^2$ ✓

$= 2\times 8 - 2\times 1$ ✓

$= 14$

(ii) $\int_0^{\frac{\pi}{2}} \sin 2x\,dx = -\frac{1}{2}\left[\cos 2x\right]_0^{\frac{\pi}{2}}$ ✓

$= -\frac{1}{2}\left[\cos\pi - \cos 0\right]$

$= -\frac{1}{2}\left[-1-1\right]$ ✓

$= 1$

QUESTION 3

(a) (i) $BC^2 = 9.81^2 + 15.24^2 - 2\times 9.81\times 15.24\times \cos 31^\circ 20'$ ✓

$= 73.089\ldots$ m

$\therefore BC \doteqdot 8.55$ m (nearest cm) ✓ (Penalise incorrect rounding but ignore no rounding to 2 d.p.)

(ii) Area $\Delta ABC = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \cdot 9.81 \cdot 15.24 \cdot \sin 31^\circ 20'$ ✓

$= 38.87\ldots$ m^2 ✓

$\doteqdot 39$ m^2

(b) (i) $r = -\frac{1}{3}$. Since $|r| < 1$, S_∞ exists ✓

(ii) $S_\infty = \frac{a}{1-r}$

$= \frac{1}{\frac{4}{3}}$

$= \frac{3}{4}$ ✓

(c) $\frac{dy}{dx} = \frac{1}{x}$

At $x = e$, $\frac{dy}{dx} = \frac{1}{e}$ ✓ (for either or both)

Equation of tangent is:

$y - 1 = \frac{1}{e}(x - e)$ ✓

$y - 1 = \frac{x}{e} - 1$

$\therefore x = ey$ ✓

(d) (i) $\alpha + \beta = -8$ ✓

(ii) $\alpha\beta = 11$ ✓

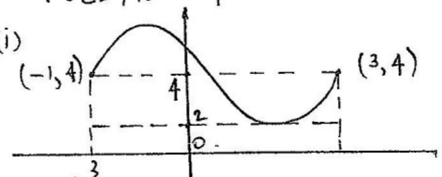
(iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (-8)^2 - 2\times 11$ ✓

$= 42$

QUESTION 4

a)(i)



✓

(ii) $\int_{-1}^{3} f(x) + 2\, dx = \frac{15}{2} + 8$ ✓

$= \frac{31}{2}$. ✓

b) $x = 2t^3 - t^2$

(i) $\dot{x} = 6t^2 - 2t$ ✓

At rest when $\dot{x} = 0$

ie, $2t(3t-1) = 0$ ✓

$\therefore t = 0$ or $\frac{1}{3}$. ✓

(ii) At $t = 0$, $x = 0$.

$t = \frac{1}{3}$, $x = 2 \cdot \frac{1}{27} - \frac{1}{9}$ ✓

$= -\frac{1}{27}$ ✓

$\therefore$ Distance travelled is $\frac{1}{27}$ m. ✓

(c) (i) $\angle ADB = 40 + \theta$ (exterior angle of Δ) ✓

(ii) $\angle DBE = 40$ (isosceles Δ)

$\angle DAE + 40 + \theta + 40 = 180$ (angle sum) ✓

$\therefore \angle DAE = 100 - \theta$ ✓

(iii) $\angle ADE = \frac{1}{2}(180 - (100 - \theta))$ (base $\angle$s equal, isosceles ΔADE) ✓

$= 40 + \frac{\theta}{2}$ ✓

$\therefore \angle EDB = (40 + \theta) - (40 + \frac{\theta}{2})$

$= \frac{\theta}{2}$ ✓

QUESTION 5

a) (i) $75^\circ = 75 \times \frac{\pi}{180}$

$= \frac{5\pi}{12}$ ✓

$\therefore l = \frac{5\pi}{12} \times 4.8$

$= 2\pi$ cm ✓

(subtract 1 mark if either answer has been approximated: i 6.283... ii 15.0796...)

(ii) Area of sector $= \frac{1}{2}.(4.8)^2.\frac{5\pi}{12}$ ✓

$= \frac{24\pi}{5}$ (or 4.8π) cm^2 ✓

(iii) Area of segment = Area of sector − Area of Δ

$= \frac{24\pi}{5} - \frac{1}{2}.(4.8)^2.\sin\frac{5\pi}{12}$ ✓

$= 3.952$ cm^2 ✓

(b) (i) $\tan x = -3$

Related angle $= 1.249$ ✓

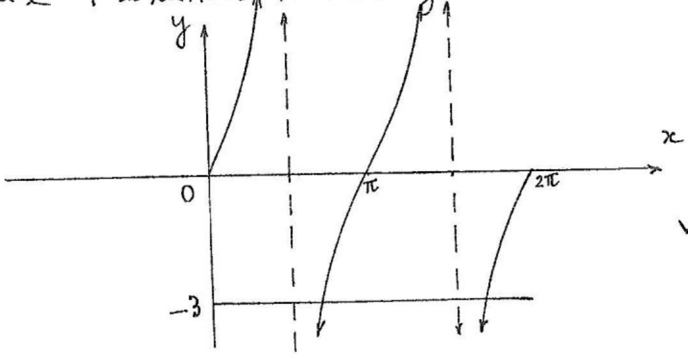
$\therefore x = \pi - 1.249$ or $2\pi - 1.249$

$= 1.893$ or 5.034 ✓

(need both for this mark. No marks for degree equivalent)

(iii) Since there are 2 solutions from $0 \le x \le 2\pi$, there are 4 solutions in the range $-2\pi \le x \le 2\pi$ ✓✓

(ii)



✓✓ (Do not penalise if they draw graphs outside domain)

QUESTION 6

a) (i) $y = 3x^2 + 2x - 1$

$\frac{dy}{dx} = 6x + 2$

At $x = -1$, $\frac{dy}{dx} = -4$ ✓

Eqtn is $y - 0 = -4(x+1)$ ✓

ie) $y + 4x + 4 = 0$

(ii) When $x = 0$, $y + 4 = 0$

$\therefore y = -4$

So $B = (0, -4)$ ✓

(iii) $M = \frac{1}{4}$ ✓

$y + 4 = \frac{1}{4}(x - 0)$

ie) $4y + 16 = x$ ✓

(iv) When $y = 0$, $x = 16$

$\therefore C = (16, 0)$ ✓

(v) Area Δ ABC $= \frac{1}{2} \times 4 \times 17$ ✓

$= 34$ units^2 ✓

(b)

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
y	0	0.17157	1

$V = \pi \int_0^{\frac{\pi}{4}} \tan^2 x \, dx$

$\doteqdot \pi \left[\frac{\pi}{24}(0 + 4 \times 0.17157 + 1) \right]$ ✓✓

$= 0.693$ (u^3) ✓ ✓

(subtract 1 mark for incorrect rounding or if they have not attempted to round off)

QUESTION 7

a) $\angle BWA = 80°$ (external $\angle$ of ΔAWD) ✓

$\angle BXC = 56°$ (external $\angle$ of ΔCXE) ✓

$x = 180 - (80 + 56)$ ($\angle$ sum of Δ)

$\therefore x = 44$ ✓

b) $\angle DPA = x$ (alternate $\angle$ on $\parallel$ lines) ✓

$\therefore \Delta ADP$ is isosceles $\Rightarrow AD = AP$ —— 1.

$\angle BPC = y$ (alternate $\angle$ on $\parallel$ lines) ✓

$\therefore \Delta BCP$ is isosceles $\Rightarrow BC = BP$ —— 2.

Since ABCD is a parallelogram, $AD = BC$. —— 3.

By 1, 2 & 3; $AD = AP = BC = BP$.

$\therefore AB = AP + BP$

$= AD + AD$

$= 2AD$, as required. ✓✓

(There are other acceptable methods. Allocate marks similarly if possible)

c) (i) $\Delta = 2^2 - 4.5.k$

$= 4 - 20k$ ✓

(ii) For real roots, $\Delta \geq 0$ ✓ (either)

ie, $4 - 20k \geq 0$

$\therefore k \leq \frac{1}{5}$ ✓

d) $\ln 16 = 2 \ln x$

ie, $2 \ln 4 = 2 \ln x$ ✓

$\therefore x = 4$ ✓

[or $\ln 16 = \ln x^2$ ✓

$\therefore x^2 = 16$

$x = \pm 4$

but $x > 0$ $\therefore x = 4$ only] ✓

QUESTION 8

a) (i) $A_1 = 1500(1.0075)$ ✓

$= \$1511.25$ ✓

(ii) $A = 1500(1.0075)^{120}$

$= \$3677.04$ ✓

(iii) Total Amount $= 1500\left[1.0075^{120} + 1.0075^{119} + 1.0075^{118} + \dots + 1.0075\right]$ ✓

$= 1500\left[1.0075 \dfrac{(1.0075^{120} - 1)}{(1.0075 - 1)}\right]$ ✓

$= 1500 \times 194.9656342$

$= \$292448$ ✓

(iv) New Total $= 1600 \times 194.9656342$

$= \$311945$ ✓

$\therefore$ Difference $= \$19497$ ✓

b) (i) Area ABCD $= \frac{1}{2} . 2\left(e + \frac{1}{e}\right)$ ✓

$= \dfrac{e^2+1}{e}$ (u^2) ✓

(ii) Area under the curve $= \displaystyle\int_{-1}^{1} e^x \, dx$ ✓

$= \left[e^x\right]_{-1}^{1}$

$= e - e^{-1}$

$= e - \frac{1}{e}$

$= \dfrac{e^2-1}{e}$ (u^2) ✓

$\therefore$ Area between the curves $= \dfrac{e^2+1}{e} - \dfrac{e^2-1}{e}$

$= \dfrac{2}{e} . (u^2)$ ✓

QUESTION 9

a) (i) $V = Ce^{-kt}$

When $t = 0$, $V = 65000$

$\therefore C = 65000$ ✓

When $t = 1$, $V = 55000$

$\therefore 55000 = 65000\, e^{-k}$

$e^{-k} = \frac{11}{13}$

$-k = \ln\left(\frac{11}{13}\right)$

$\therefore k = -\ln\left(\frac{11}{13}\right)$

$\doteqdot 0 \cdot 16705\ldots$ } ✓ (either)

(ii) When $t = 5$,

$V = 65000\, e^{-5k}$

$= \$28194$ ✓

(iii) We need t such that

$V < \frac{65000}{2}$

ie/ $65000\, e^{-kt} < \frac{65000}{2}$

$e^{-kt} < \frac{1}{2}$

$-kt < \ln \frac{1}{2}$

$t > 4 \cdot 149\ldots$ } ✓ (any)

$\therefore$ Car falls below half its cost price in 2005 ✓

b) (i) $R = 9t^2 - t^4$

When $t = 2$; $R = 9.4 - 2^4$

$= 20$ m^3/h ✓

(ii) When $t = 3$, $R = 0$ and when $t > 3$, $R < 0$ which suggests that concrete is going back into the truck! Since $t \geqslant 0$ we have $0 \leqslant t \leqslant 3$ ✓

(iii) $\frac{dR}{dt} = 18t - 4t^3$ and $\frac{d^2R}{dt^2} = 18 - 12t$ ✓

$= 2t(9 - 2t^2)$ ✓

$\frac{dR}{dt} = 0$ when $t = 0$ or $\frac{3}{\sqrt{2}}$ or $-\frac{3}{\sqrt{2}}$

Since $t \geqslant 0$, ignore $t = -\frac{3}{\sqrt{2}}$. When $t = 0$, $\frac{d^2R}{dt^2} > 0$ and when $t = \frac{3}{\sqrt{2}}$, $\frac{d^2R}{dt^2} < 0$.

✓ So maximum flow-rate occurs when $t = \frac{3}{\sqrt{2}}$. ie/ $R = 9 . \frac{9}{2} - \frac{81}{4}$

$= \frac{81}{4}$ m^3/h ✓

(iv) $A = \int 9t^2 - t^4\, dt$

$A = 3t^3 - \frac{t^5}{5} + c$ ✓

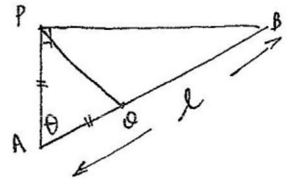
When $t = 0$, $A = 1000$ $\therefore c = 1000$.

$\therefore A = 3t^3 - \frac{t^5}{5} + 1000$. ✓

(Be generous in part (ii).

QUESTION 10

a)



(i) $\cos\theta = \frac{AP}{l}$

$\therefore AP = l \cos\theta$ ✓

$S = \frac{1}{2} . l\cos\theta . l\cos\theta . \sin\theta$ ✓

$S = \frac{l^2}{2} . \cos^2\theta . \sin\theta$, as required. ✓

$= \frac{l^2}{2} . (1 - \sin^2\theta)(\sin\theta)$

$= \frac{l^2}{2} (\sin\theta - \sin^3\theta)$,

(ii) $\frac{dS}{d\theta} = \frac{l^2}{2}\left(\cos\theta - 3\sin^2\theta\cos\theta\right) = \frac{l^2}{2}\cos\theta\left(1 - 3\sin^2\theta\right)$ ✓

$\frac{dS}{d\theta} = 0$ when $\cos\theta = 0$ or $\sin\theta = \pm\frac{1}{\sqrt{3}}$

since $0 < \theta < 90^\circ$, $\sin\theta = \frac{1}{\sqrt{3}}$ is only possible solution.

$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$\frac{3}{4}$
$\frac{dS}{d\theta}$	+	0	−

$\therefore$ When $\sin\theta = \frac{1}{\sqrt{3}}$, S is a maximum. ✓

$S = \frac{l^2}{2} . \left(\frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}}\right)$

$= \frac{l^2}{2} . \left(\frac{3 - 1}{3\sqrt{3}}\right)$

$= \frac{l^2}{3\sqrt{3}}$

$= \frac{\sqrt{3}\, l^2}{9}$ is the maximum area. ✓

b) (i) $\frac{1}{(x-1)\,x} = \frac{Ax + B(x-1)}{(x-1).\,x}$

$\therefore \quad 1 = Ax + Bx - B$

$B = -1$ and $A + B = 0$

$\therefore A = 1$ ✓✓

$\therefore \frac{1}{(x-1)\,x} = \frac{1}{x-1} - \frac{1}{x}$

(ii) $S_n = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \cdots + \frac{1}{(n-1)\times n}$

$= \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{n-1} - \frac{1}{n}\right)$ ✓

$= \frac{1}{1} + \left(-\frac{1}{2} + \frac{1}{2}\right) + \left(-\frac{1}{3} + \frac{1}{3}\right) + \cdots + \left(\frac{-1}{n-1} + \frac{1}{n-1}\right) - \frac{1}{n}$ ✓

$= 1 - \frac{1}{n}$, as required.

(iii) $\sum_{n=2}^{\infty} \frac{1}{(n-1)\,n} = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \cdots$

$= \lim_{n\to\infty} S_n$ (, from (ii)) ✓

$= \lim_{n\to\infty} \left(1 - \frac{1}{n}\right)$

$= 1.$ ✓